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# The $K_L \to \pi^0 \nu \bar{\nu}$ decay in models of extended scalar sector

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Abstract: We calculate new contributions to the $K_L \to \pi^0 \nu \bar{\nu}$ decay in models where neutrino Majorana masses require an extension of the scalar sector. First, we study a model where the neutrino mass is induced by the vacuum expectation value of an SU(2)-triplet scalar. Second, we study the Zee model where the Majorana mass comes from one loop diagrams involving a singly charged, SU(2)-singlet scalar. In both models, the Yukawa couplings that involve the new scalar and the neutrinos could be of order one. We find, however, that the contributions to the $K_L \to \pi^0 \nu \bar{\nu}$ decay mediated by the new scalars depend on the neutrino masses rather than the Yukawa couplings and are, therefore, negligibly small.

Keywords: Neutrino Physics, Rare Decays, Kaon Physics, Beyond Standard Model.

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## Contents



# 1. Introduction

Within the standard model (SM) the $K_L \to \pi^0 \nu\bar{\nu}$ decay is known to be a CP violating (CPV) process to a very good approximation [1, 2], and subject to a clean theoretical interpretation [3]. In the SM, CP conserving (CPC) contributions are chirally suppressed and smaller by many orders of magnitude than the CPV ones [2]. In a previous work [4], we calculated the CPC contributions that arise when the SM lagrangean is extended to include neutrino mass terms. These contributions are known to be chirally enhanced, but we found that they are suppressed by a factor of order $m_\nu^2/m_W^2$ which makes them negligibly small. In this work we study models where, in addition to neutrino mass terms, there are new light scalars. The question that we ask is whether the fact that such scalars can have Yukawa couplings of order one to neutrinos allows for a situation where the new contributions are chirally enhanced while avoiding the $m_\nu^2/m_W^2$ suppression factor.

Specifically, we consider the following two models:

1. Neutrino masses are related to the vacuum expectation value of an SU(2)-triplet scalar. We present the model and calculate the new contributions to the $K_L \to \pi^0 \nu\bar{\nu}$ decay in section 2.

2. Neutrino masses are related to loop diagrams involving a singly charged SU(2)-singlet and an additional SU(2)-doublet scalar. We present the model and calculate the new contributions to the $K_L \to \pi^0 \nu\bar{\nu}$ decay in section 3.

A summary of our conclusions is given in section 4.

## 2. An SU(2) triplet scalar

### 2.1 The model

We consider the SM lagrangean with the addition of an SU(2) triplet Higgs field ($\Delta$). The most general scalar potential is given by [5]:

$$
\begin{aligned}
V(H,\Delta) = {} & \frac{\lambda_H}{2}(H_i^\dagger H^i)^2 + \frac{\lambda_\Delta}{2}(\Delta_t^\dagger \Delta^t)^2 + \lambda H_i^\dagger H^i \Delta_t^\dagger \Delta^t + \widetilde{\lambda} H^{i\dagger}\sigma_{ij}^q H^j \Delta^{r\dagger} t_{rs}^q \Delta^s + \\
& + m\left(\bar{\Delta}^0 H^0 H^0 + \sqrt{2}\Delta^- H^+ H^0 + \Delta^{--} H^+ H^+\right) + \\
& + \mu_H^2 H_i^\dagger H^i + \mu_\Delta^2 \Delta_t^\dagger \Delta^t + \text{h.c.}\,,
\end{aligned}
\tag{2.1}
$$

where $\Delta^t = [\Delta^{++}, \Delta^+, \Delta^0]$, $H^i = [H^+, H^0]$ and $\sigma, t$ are are SU(2) generators in the $J = 1/2, 1$ representations, respectively. The $m$-term in (2.1) breaks the global lepton number symmetry ($L$) so that the phenomenologically unacceptable [6, 7] Majoron [8] is avoided. For simplicity, we assume that the couplings in (2.1) conserve CP. As concerns the VEVs, $\langle H^0 \rangle \equiv v_H/\sqrt{2}$ and $\langle \Delta^0 \rangle \equiv v_\Delta/\sqrt{2}$, we assume for simplicity that they are both real and we take into account the constraints from the $\rho$ parameter [8, 9]:

$$
\frac{v_\Delta}{v_H} \lesssim 10^{-2}\,. \tag{2.2}
$$

In order to calculate the contribution to the decay, the scalar mass eigenstates and mixing angles should be identified. With the assumption that the scalar potential $V(H,\Delta)$ (2.1) is real, the imaginary and the real parts of the neutral scalars remain unmixed. For the CP even fields, $\sqrt{2}\,\mathrm{Re}(H^0)$ and $\sqrt{2}\,\mathrm{Re}(\Delta^0)$, we have the following mass matrix [5] (we neglect terms that are higher order in $v_\Delta/v_H$ and assume that the $\lambda$'s are positive and of order one):

$$
M_{\mathcal{R}}^2 = v_H^2 \begin{pmatrix} \lambda_H & \frac{\sqrt{2}}{v_H}(\hat{\lambda}\frac{v_\Delta}{\sqrt{2}} + m) \\ \frac{\sqrt{2}}{v_H}(\hat{\lambda}\frac{v_\Delta}{\sqrt{2}} + m) & -\frac{m}{\sqrt{2}v_\Delta} \end{pmatrix}, \tag{2.3}
$$

where $\hat{\lambda} \equiv \lambda + \widetilde{\lambda}$. The eigenvectors of $M_{\mathcal{R}}^2$ are:

$$
\begin{aligned}
\xi_1 &= \mathrm{Re}(H^0)\cos\gamma + \mathrm{Re}(\Delta^0)\sin\gamma \\
\xi_2 &= -\,\mathrm{Re}(H^0)\sin\gamma + \mathrm{Re}(\Delta^0)\cos\gamma\,,
\end{aligned}
\tag{2.4}
$$

where

$$
\begin{aligned}
\tan\gamma &= -\frac{v_H}{2v_\Delta}\left(\hat{\lambda} + \frac{\sqrt{2}m}{v_\Delta}\right)^{-1}\left[\lambda_H + \frac{m}{\sqrt{2}v_\Delta} - \sqrt{\left(\lambda_H + \frac{m}{\sqrt{2}v_\Delta}\right)^2 + 4\frac{v_\Delta^2}{v_H^2}\left(\hat{\lambda} + \frac{\sqrt{2}m}{v_\Delta}\right)^2}\,\right] \\
&\approx 2\frac{v_\Delta}{v_H}\left(\frac{\hat{\lambda} + \sqrt{2}m/v_\Delta}{2\lambda_H + \sqrt{2}m/v_\Delta}\right) \ll 1\,.
\end{aligned}
\tag{2.5}
$$



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In deriving the inequality in eq. (2.5) and below we assume that there are no fine-tuned cancellations among the independent parameters of the scalar sector. The eigenvectors correspond to the following mass eigenvalues:

$$
\begin{aligned}
m^2_{\xi_{1,2}} &= \frac{v_H^2}{2}\left[\lambda_H - \frac{m}{\sqrt{2}v_\Delta} \pm \sqrt{\left(\lambda_H + \frac{m}{\sqrt{2}v_\Delta}\right)^2 + 4\frac{v_\Delta^2}{v_H^2}\left(\hat{\lambda} + \frac{\sqrt{2}m}{v_\Delta}\right)^2}\,\right] \\
&\approx v_H^2\left[\lambda_H + \frac{v_\Delta^2}{v_H^2}\frac{\left(\hat{\lambda} + \sqrt{2}m/v_\Delta\right)^2}{\lambda_H + m/\sqrt{2}v_\Delta}, -\frac{m}{\sqrt{2}v_\Delta} + O\left(m\frac{v_\Delta^2}{v_H^2}\right)\right]. \qquad (2.6)
\end{aligned}
$$

For the CP odd fields, $\sqrt{2}\,\mathrm{Im}(H^0)$ and $\sqrt{2}\,\mathrm{Im}(\Delta^0)$, we have the following mass matrix:

$$
M_{\mathcal{I}}^2 = \sqrt{2}m\begin{pmatrix} -2v_\Delta & -v_H \\ -v_H & -\frac{v_H^2}{2v_\Delta}\end{pmatrix}. \qquad (2.7)
$$

The eigenvectors of $M_{\mathcal{I}}^2$ are:

$$
\begin{aligned}
G^0 &= \mathrm{Im}(H^0)\cos\eta + \mathrm{Im}(\Delta^0)\sin\eta \\
J^0 &= -\,\mathrm{Im}(H^0)\sin\eta + \mathrm{Im}(\Delta^0)\cos\eta\,, \qquad (2.8)
\end{aligned}
$$

where

$$
|\tan\eta| = \left|\frac{2v_\Delta}{v_H}\right| \ll 1\,. \qquad (2.9)
$$

We learn that mixing in the CP-odd sector is very small and we neglect it from here on. The eigenvectors correspond to the following mass eigenvalues [5]:

$$
m^2_{G^0,J^0} = \left[0, -\frac{mv_H^2}{\sqrt{2}v_\Delta}\right], \qquad (2.10)
$$

where the massless component $G^0 \sim \sqrt{2}\,\mathrm{Im}(H^0)$ corresponds to the unphysical Goldstone boson which is eaten by the $Z$ boson, while $J^0 \sim \sqrt{2}\,\mathrm{Im}(\Delta^0)$ corresponds to the would-be Majoron. As expected its squared mass is proportional to the explicit lepton number violating parameter $m$.

For the singly charged scalars, $H^\pm$ and $\Delta^\pm$, we have the following mass matrix:

$$
M_C^2 = \frac{v_H}{\sqrt{2}}\left(\tilde{\lambda}\frac{v_\Delta}{\sqrt{2}} + m\right)\begin{pmatrix} -2\frac{v_\Delta}{v_H} & \sqrt{2} \\ \sqrt{2} & -\frac{v_H}{v_\Delta}\end{pmatrix}. \qquad (2.11)
$$

The eigenvectors of $M_C^2$ are:

$$
\begin{aligned}
G^\pm &= H^\pm\cos\eta' + \Delta^\pm\sin\eta' \\
\xi^\pm &= -H^\pm\sin\eta' + \Delta^\pm\cos\eta'\,, \qquad (2.12)
\end{aligned}
$$

where

$$|\tan\eta'| = \left|\frac{\sqrt{2}v_\Delta}{v_H}\right| \ll 1\,. \tag{2.13}$$

We learn that mixing in the charged sector is very small and we neglect it from here on. The eigenvectors correspond to the following mass eigenvalues:

$$m^2_{G^\pm,\xi^\pm} = \left[0, -\frac{v_H^2}{\sqrt{2}}\left(\frac{\widetilde{\lambda}}{\sqrt{2}} + \frac{m}{v_\Delta}\right)\right], \tag{2.14}$$

where the massless component $G^\pm \sim H^\pm$ corresponds to the unphysical charged Goldstone boson which is eaten by the $W^\pm$ bosons, while $\xi^\pm \sim \Delta^\pm$ corresponds to a new physical charged scalar.

Neutrino Majorana masses are induced via the following interaction terms:

$$\mathcal{L}_{\nu\nu} = f_{mn}(L^m_{Li})_\alpha \tau^2_{\alpha\beta}\sigma^2_{ir}\sigma^t_{rj}(L^n_{Lj})_\beta \Delta^t\,, \tag{2.15}$$

where $\tau$ is an SU(2) generator in the $J = 1/2$ spinor representation. For simplicity, we take $f_{mn} = f_m\delta_{mn}$ with $f_m$ real. In this way, we avoid unnecessary complications related to flavor mixing and CP violation in the neutrino sector (see discussion in ref. [4]). The interaction term (2.15) induces neutrino masses:

$$(m_\nu^{\rm Maj})_i = f_i\langle\Delta^0\rangle \tag{2.16}$$

with $i = 1, 2, 3$. In addition, it generates new contributions to the $K_L \to \pi^0\nu\bar{\nu}$ decay.

## 2.2 The $K_L \to \pi^0\nu\bar{\nu}$ decay rate

The dominant new contributions to the $K_L \to \pi^0\nu\bar{\nu}$ decay come from the diagrams presented in figure 1, which generate the following effective hamiltonian:

$$\mathcal{H}^\Delta_{\rm eff} = \sum_\ell \frac{G_F}{\sqrt{2}}\frac{\alpha}{2\pi\sin^2\Theta_W}(\bar{d}s)(\nu_\ell^T i\tau^2\nu_\ell)\left(X_\ell^{(a)} + X_\ell^{(b)} + X_\ell^{(c)}\right), \tag{2.17}$$

where the $X_\ell^{(i)}$ function corresponds to the diagram in figure 1$i$. In the $X_\ell^{(a)}$ and $X_\ell^{(b)}$ terms, the L breaking effects enter through, respectively, the $\Delta WW$ and $\Delta HH$ couplings and is related to the spontaneous breaking ($\langle\Delta^0\rangle \neq 0$):

$$\begin{aligned} X_\ell^{(a)} + X_\ell{}^{(b)} = & -m_\nu^\ell m_s \sum_i \lambda_i \left(\frac{\sin^2\gamma}{m^2_{\xi_1}} + \frac{\cos^2\gamma}{m^2_{\xi_2}}\right)\left(\frac{1}{2} + 2x_i\frac{\lambda - \widetilde{\lambda}}{g^2}\right) \times \\ & \times \left[1 + \frac{x_i}{(x_i-1)^3}\left(2x_i\ln(x_i) + 1 - x_i^2\right)\right], \end{aligned} \tag{2.18}$$

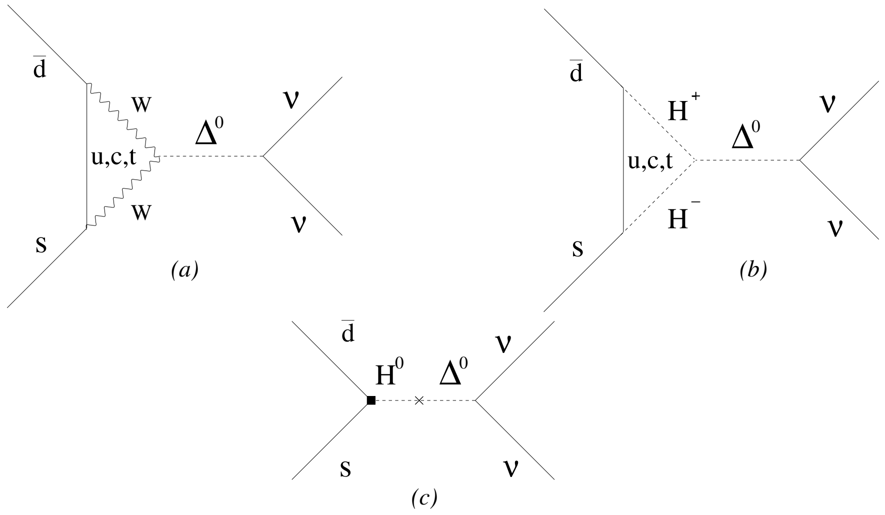


**Figure 1:** CPC penguin diagrams mediated by a triplet Higgs.

where $x_i = (m_i/M_W)^2$ and $\lambda_i = V_{is}^* V_{id}$. Since the top quark contribution is dominant, (2.18) can be simplified:

$$X_\ell^{(a)} + X_\ell^{(b)} \approx -\lambda_t m_\nu^\ell m_s \left( \frac{\sin^2\gamma}{m_{\xi_1}^2} + \frac{\cos^2\gamma}{m_{\xi_2}^2} \right) \times$$
$$\times x_t \left[ \left( \frac{1}{2} + 2x_t \frac{\lambda - \widetilde{\lambda}}{g^2} \right) \frac{(2x_t \ln(x_t) + 1 - x_t^2)}{(x_t - 1)^3} + 2\frac{\lambda - \widetilde{\lambda}}{g^2} \right] . \tag{2.19}$$

In the $X_\ell^{(c)}$ term, the L breaking effect enters through the $\Delta - H$ mixing and is related to both the soft breaking ($m \neq 0$) and the nonzero VEV of $\Delta^0$. The calculation of $X_\ell^{(c)}$ is simplified by the use of the $sdH$ effective coupling $\Gamma_{sdH}$, (represented by a square in figure 1$c$) which was calculated within the SM in ref. [10]. For our model $\Gamma_{sdH}$ should be expressed in terms of the appropriate masses and mixing angles. The mixing of the charged scalar can, however, be neglected (see eq. (2.13)) and therefore we use directly the SM calculation:

$$\Gamma_{sdH} = -\lambda_t \frac{g^3}{128\pi^2} \frac{m_t^2 m_s}{M_W^3} \left( \frac{3}{2} + \frac{4\lambda_H}{g^2} f_2(x_t) \right) (1 + \gamma^5) , \tag{2.20}$$

where

$$f_2(x) = \frac{x}{2(1-x)^2} \left( -\frac{x}{1-x} \ln x + \frac{2}{1-x} \ln x - \frac{1}{2} - \frac{3}{2x} \right) . \tag{2.21}$$

Then $X_\ell^{(c)}$ is given by

$$X_\ell^{(c)} = -\lambda_t m_\nu^\ell m_s \frac{\sin 2\gamma}{8} \frac{v_H}{v_\Delta} \left( \frac{1}{m_{\xi_1}^2} - \frac{1}{m_{\xi_2}^2} \right) x_t \left( \frac{3}{2} + \frac{4\lambda_H}{g^2} f_2(x_t) \right) . \tag{2.22}$$

Note that since the lepton and quark operators in the effective hamiltonian $\mathcal{H}_{\text{eff}}^\Delta$ in eq. (2.17) are scalar operators, the new contributions are CPC.

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We are interested in finding the upper bound on the new contributions. We therefore focus on the region in parameter space that maximizes them. For all scalar masses, we will use a lower bound of 45 GeV, thus avoiding any conflict with constraints from the invisible width of the $Z$ boson. From eqs. (2.10) and (2.14) we then find:

$$\mathrm{sign}\left(\frac{m}{v_\Delta}\right) = -1\,, \qquad \left|\frac{m}{v_\Delta}\right| \gtrsim 1\,. \tag{2.23}$$

Substituting the proper values for the masses and angles (2.5), (2.6) into the functions $X_\ell^{(a)}, X_\ell^{(b)}$ (2.18) and $X_\ell^{(c)}$ (2.22) we find:

$$\begin{aligned} X_\ell^{(a)} + X_\ell^{(b)} &\sim \lambda_t \frac{m_\nu^\ell m_s}{v_H^2}\left(\frac{v_\Delta}{m} + O\left(\frac{v_\Delta^2}{v_H^2}\right)\right) \\ X_\ell^{(c)} &\sim \lambda_t \frac{m_\nu^\ell m_s}{v_H^2}\left(\frac{v_H^2}{\lambda_H v_H^2 + m v_\Delta} + O\left(\frac{v_\Delta^2}{v_H^2}\right)\right). \end{aligned} \tag{2.24}$$

For $m \gg v_H$ $X_\ell^{(a)} + X_\ell^{(b)}$ and $X_\ell^{(c)}$ are all highly suppressed, together with the total rate. Larger contributions are found with $m$ in the intermediate regime $v_\Delta \lesssim m \lesssim v_H$, where $X_\ell^{(a)} + X_\ell^{(b)}$ and $X_\ell^{(c)}$ are all of the order of $m_\nu^\ell m_s / v_H^2 \sim m_\nu^\ell m_s / m_{\xi_{1,2}}^2$.

The $\nu_m \nu_m \Delta$ coupling, that is $f_m$ of eq. (2.15), is proportional to $m_{\nu_m}/v_\Delta$. Naively, one may think that this mechanism of inducing neutrino masses can induce a contribution to the $K_L \to \pi\nu\nu$ rate that is enhanced by a factor of order $v_H/v_\Delta$ compared to the mechanisms of [4]. We find that this is not the case. For the diagrams in figure 1*a,b*, there is a $v_\Delta$-factor in the $WW\Delta$ and $HH\Delta$ couplings. For the diagram in figure 1*c*, there is a factor of $\sin 2\gamma v_H (1/m_{\xi_1}^2 - 1/m_{\xi_2}^2) \sim v_\Delta / v_H^2$. In either case, the final result is proportional to $f_m v_\Delta \sim m_{\nu_m}$ and there is no enhancement.

We are now in a position to compare the contribution of the triplet scalar to the leading, CPV one [11]:

$$R_{\mathrm{CPV}}^\Delta = \frac{\Gamma_\Delta(K_L \to \pi^0 \nu\bar\nu)}{\Gamma_{\mathrm{CPV}}(K_L \to \pi^0 \nu\bar\nu)} \lesssim \left(\frac{M_K}{m_{\xi_{\mathrm{light}}}}\frac{m_\nu}{m_{\xi_{\mathrm{light}}}}\right)^2 \approx 10^{-11}\left[\frac{m_\nu}{10\,\mathrm{MeV}}\right]^2\left[\frac{45\,\mathrm{GeV}}{m_{\xi_{\mathrm{light}}}}\right]^4, \tag{2.25}$$

where $m_{\xi_{\mathrm{light}}}$ stands for the lighter between $m_{\xi_1}$ and $m_{\xi_2}$. While the direct bound is $m_{\nu_\tau} \leq 18.2\,\mathrm{MeV}$ [7], there is a significantly stronger bound from cosmology, $m_\nu \lesssim 10\,\mathrm{eV}$ [12]. Therefore, very likely, $R_{\mathrm{CPV}}^\Delta \lesssim 10^{-23}$.

## 3. The Zee model

### 3.1 The model

The Zee model [13] enlarges the SM scalar sector by a charged SU(2) singlet $\phi^+$ and an SU(2) doublet $H_2$. The SM doublet is denoted by $H_1$. The lagrangean of the model is [13]:

$$\mathcal{L}_{\mathrm{Zee}} \equiv \mathcal{L}_{SM} + f_{mn}\epsilon_{\alpha\beta}(L^{m\alpha T} C L^{n\beta}{}_L)\phi^+ + \mu(H_1^\dagger H_2 + H_2^\dagger H_1) + V(H_1, H_2, \phi^+)\,, \tag{3.1}$$

where $L$ is a lepton doublet, $\alpha, \beta$ are SU(2) indices, $m, n$ are flavor indices, $C$ is the Dirac charge conjugation matrix and $V(H_1, H_2, \phi^+)$ is the most general scalar potential which respects the SM gauge symmetries and $L$ [14]:

$$\begin{aligned} V(H_1, H_2, \phi^+) \equiv\ & \lambda_1 \left( |H_1|^2 - \frac{a_1^2}{2} \right)^2 + \lambda_2 \left( |H_2|^2 - \frac{a_2^2}{2} \right)^2 + \\ & + \lambda_3 \left( |H_1|^2 - \frac{a_1^2}{2} \right) \left( |H_2|^2 - \frac{a_2^2}{2} \right) + \\ & + \lambda_4 (|H_1|^2 |H_2|^2 - {H_1}^\dagger H_2 {H_2}^\dagger H_1) + \lambda_5 \left| {H_1}^T i\tau^2 H_2 \right|^2 + \\ & + \lambda_6 m^2 \left|\phi^+\right|^2 + \lambda_7 \left|\phi^+\right|^4 + \lambda_8 \left|\phi^+\right|^2 |H_1|^2 + \\ & + \lambda_9 \left|\phi^+\right|^2 |H_2|^2 + M \left( {H^1}^T i\tau^2 H^2 \phi^- + \text{h.c.} \right) . \end{aligned} \tag{3.2}$$

Both Higgs doublets develop nonzero VEVs:

$$\langle {H^0}_{1,2} \rangle \equiv \frac{v_{1,2}}{\sqrt{2}} . \tag{3.3}$$

For $\mu = 0$, the parameters $a_{1,2}$ would be equal to $v_{1,2}$ respectively. With a nonzero $\mu$ term, $v_{1,2}$ are complicated functions of $a_i$, $\lambda_i$ and $\mu$. The $L$-charges of the scalars fields are:

$$L(H_1) = 0 , \qquad L(H_2) = 2 , \qquad L(\phi^+) = -2 . \tag{3.4}$$

Then we see that $L$ is broken spontaneously via the nonzero VEV of $H_2$. The $\mu$-term in eq. (3.1) breaks $L$ explicitly so that the phenomenologically unacceptable Majoron [7, 14, 15] is avoided. For simplicity, we assume that CP is conserved in the lepton sector and take all the dimensionless scalar couplings to be of order one.

In order to calculate the new contribution to the decay, the scalar mass eigenstates and mixing angles should be identified. There are seven physical and three unphysical combinations (eaten by the $Z$ and the $W^\pm$ bosons). Out of the seven physical combinations, four are charged and three are neutral. The neutral ones are the would-be Majoron (which gains mass due to the $\mu$ term in eq. (3.1)) and two other real fields. As long as $\mu$ is real, the real and imaginary parts of $H^0_{1,2}$ remain unmixed (as in the cases in which L is spontaneously broken [14, 16]), thus the imaginary physical combination corresponds to the would-be Majoron which is irrelevant to our calculation.

The real neutral mass matrix is read from the scalar potential of eq. (3.1):

$$M_{\mathcal{R}} = \frac{1}{2} \begin{pmatrix} 2\lambda_1 (3{v_1}^2 - {a_1}^2) + \lambda_3 ({v_2}^2 - {a_2}^2) & 2\lambda_3 v_1 v_2 + \mu \\ 2\lambda_3 v_1 v_2 + \mu & 2\lambda_2 (3{v_2}^2 - {a_2}^2) + \lambda_3 ({v_1}^2 - {a_1}^2) \end{pmatrix} . \tag{3.5}$$

The eigenvectors of $M_{\mathcal{R}}$ are:

$$\begin{aligned} \rho_1 &= \mathrm{Re}(H^0_1) \cos\theta + \mathrm{Re}(H^0_2) \sin\theta \\ \rho_2 &= -\,\mathrm{Re}(H^0_1) \sin\theta + \mathrm{Re}(H^0_2) \cos\theta . \end{aligned} \tag{3.6}$$

Each eigenvector corresponds to an eigenvalue denoted by $m_{\rho_{1,2}}$. The masses $m_{\rho_{1,2}}$ and $\tan\theta$ are complicated functions of $a_i$, $\lambda_i$ and $\mu$.



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The charged mass matrix is:

$$M_C = \frac{1}{2}\begin{pmatrix} \bar{\lambda}v_2^2 - 2\mu\frac{v_2}{v_1} & -\bar{\lambda}v_1v_2 + 2\mu & \sqrt{2}Mv_2 \\ -\bar{\lambda}v_1v_2 + 2\mu & \bar{\lambda}v_1^2 - 2\mu\frac{v_1}{v_2} & -\sqrt{2}Mv_1 \\ \sqrt{2}Mv_2 & -\sqrt{2}Mv_1 & 2\lambda_6 m^2 + \lambda_8 v_1^2 + \lambda_9 v_2^2 \end{pmatrix}, \tag{3.7}$$

where $\bar{\lambda} \equiv \lambda_4 + \lambda_5$. The massless combination of the charged fields (see e.g. refs. [17, 18]) is:

$$G^{\pm} = H_1^{\pm}\cos\delta + H_2^{\pm}\sin\delta\,, \tag{3.8}$$

with

$$\tan\delta = \frac{v_2}{v_1}\,. \tag{3.9}$$

The massless combination is not affected by the addition of the singlet field $\phi^+$, nor by the $\mu$ term, since it is determined (according to the Goldstone theorem) only by the broken $\mathrm{SU}(2)_L$ generators. The physical charged fields are given by the following linear combinations of $H_{1,2}^{\pm}$ and $\phi^{\pm}$ (which must be orthogonal to $G^{\pm}$):

$$\begin{aligned} \chi_1^{\pm} &= (-H_1^{\pm}\sin\delta + H_2^{\pm}\cos\delta)\cos\beta + \phi^{\pm}\sin\beta \\ \chi_2^{\pm} &= -(-H_1^{\pm}\sin\delta + H_2^{\pm}\cos\delta)\sin\beta + \phi^{\pm}\cos\beta\,, \end{aligned} \tag{3.10}$$

with $\tan\beta$ being a complicated function of the scalar potential parameters. The new interactions in eq. (3.1) induced a neutrino Majorana mass matrix $M^{\nu}_{m\ell}$. For simplicity we assume the dominance of the one loop induced mass [13, 14, 16, 19]:

$$M^{\nu}_{m\ell} = \frac{2\sqrt{2}}{(4\pi)^2} f_{m\ell}\tan\delta\sin 2\beta\frac{(m_\ell^2 - m_m^2)}{M_W}\ln\left(\frac{m_{\chi_2}}{m_{\chi_1}}\right), \tag{3.11}$$

with $m, \ell$ flavor indices.

### 3.2 The $K_L \to \pi^0\nu\bar{\nu}$ decay rate

The new interactions also generate new contributions to the $K_L \to \pi^0\nu\bar{\nu}$ decay. Note that the one loop induced mass and the new contributions to the decay (shown in figure 2) are related to mixing between the charged scalars and therefore vanish in the

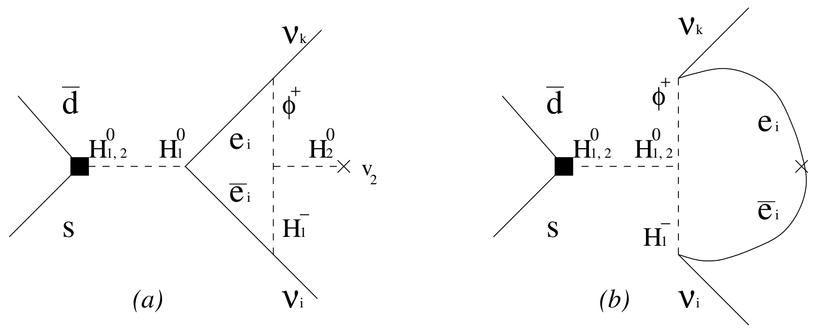


**Figure 2:** CPC diagrams in the Zee model.

limit $\delta, \beta \to 0$. Below we concentrate on the contributions which are dominant when $\sin\delta$ is small. Contributions that are of higher order do not modify significantly the results, even with $\sin\delta \sim 1$, and thus they are omitted. Note that, since $v_1$ induces the top mass, we always have $\tan\delta \lesssim 1$.

The scalar operator is induced by the neutral Higgs mediated penguin diagram shown in figure 2. The square in the figures represents an effective $sdH_{1,2}$ vertex denoted by $\Gamma_{sdH_{1,2}}$, similar to the one we encountered in section 2.2. The effective vertices $\Gamma_{sdH_{1,2}}$ induced by the Zee model fields differ from the one calculated within the SM [10]. Since, however, we are only interested in finding an upper bound on the decay rate, we can simply set the charged mixing angles factors to one, and replace the boson masses in the propagator (as explained in section 2.2) with $m_{\chi_{\rm light}} \sim 45\,\mathrm{GeV}$. Then we get:

$$\Gamma_{sdH_{1,2}} \lesssim \frac{g^3\lambda_t}{128\pi^2}\frac{m_t^2 m_s}{m_{\chi_{\rm light}}^3}(1+\gamma^5)\times\left[C_{sdH_1}, C_{sdH_2}\right], \tag{3.12}$$

where $C_{sdH_i}$ is a constant of $O(10)$. Neglecting subdominant contributions of $O[(m_\ell/m_{\rho_i,\chi_i})^4]$, the diagrams in figure 2 generate the following CPC effective hamiltonian:

$$\mathcal{H}_{\rm eff}^{\phi} = \frac{G_F}{\sqrt{2}}\frac{\alpha}{2\pi\sin^2\Theta_W}\lambda_t(\bar{d}s)(\nu_m^T i\sigma^2\nu_{L\ell})(Y_{\ell m}^{(a)}+Y_{\ell m}^{(b)}), \tag{3.13}$$

where the $Y_{\ell m}^{(i)}$ function corresponds to the diagram in figure $2i$. $Y_{\ell m}^{(a)}$ is given by:

$$Y_{\ell m}^{(a)} = 2\frac{M_{\ell m}^{\nu}M_W}{\cos\delta}\frac{m_t^2 m_s}{m_{\chi_{\rm light}}^3}\left[C_{sdH_1}\left(\frac{\cos^2\theta}{m_{\rho_1}^2}+\frac{\sin^2\theta}{m_{\rho_2}^2}\right)+C_{sdH_2}\frac{\sin 2\theta}{2}\left(\frac{1}{m_{\rho_1}^2}-\frac{1}{m_{\rho_2}^2}\right)\right]. \tag{3.14}$$

The calculation of the diagram in figure $2b$ is much more involved. This is due to the fact that each of the trilinear couplings $H_i^0 H_j^{\pm} H_k^{\mp}$ and $H_i^0 H_j^{\pm}\phi^{\mp}$ $(i,j,k=1,2)$ translates into a set of eight couplings in the mass basis. In order to estimate the upper bound on the rate we set (again) all mixing factors to unity and replace the charged scalar propagators by their maximal values, i.e.

$$\frac{1}{k^2-m_{\chi_1}^2}-\frac{1}{k^2-m_{\chi_2}^2}\to\frac{1}{k^2-m_{\chi_{\rm light}}^2}$$

and

$$\frac{1}{k^2-m_{\chi_1}^2}+\frac{1}{k^2-m_{\chi_2}^2}\to\frac{2}{k^2-m_{\chi_{\rm light}}^2}.$$

We then find:

$$\begin{aligned} Y_{\ell m}^{(b)} &\lesssim \frac{\sqrt{2}}{g64\pi^2}f_{\ell m}(m_\ell^2-m_m^2)\frac{M_W^2}{m_{\rho_1}^2}\frac{m_t^2 m_s}{m_{\chi_{\rm light}}^5}\left[F_1(\theta,m_{\rho_i},m_{\chi_i},M)+F_2(\theta,m_{\rho_i},m_{\chi_i},M)\right] \\ &= \frac{1}{8g}\frac{\cot\delta}{\sin 2\beta}\left(\ln\frac{m_{\chi_2}}{m_{\chi_1}}\right)^{-1}\frac{M_{m\ell}^{\nu}m_s}{m_{\rho_1}^2}\frac{m_t^2 M_W^3}{m_{\chi_{\rm light}}^5}\times \\ &\quad\times\left[F_1(\theta,m_{\rho_i},m_{\chi_i},M)+F_2(\theta,m_{\rho_i},m_{\chi_i},M)\right], \end{aligned} \tag{3.15}$$

where

$$F_1(\theta, m_{\rho_i}, m_{\chi_i}, M) = \left[C_{sdH_1}\left(\cos^2\theta + \sin^2\theta\frac{m^2_{\rho_1}}{m^2_{\rho_2}}\right) + C_{sdH_2}\frac{\sin 2\theta}{2}\left(-1 + \frac{m^2_{\rho_1}}{m^2_{\rho_2}}\right)\right] \times$$
$$\times \left(2\lambda_1 + \frac{1}{2}\bar{\lambda} + 2\lambda_3 + 2\lambda_8 + 3\frac{M}{v_1}\right) , \tag{3.16}$$

and

$$F_2(\theta, m_{\rho_i}, m_{\chi_i}, M) = \left[C_{sdH_2}\left(\sin^2\theta + \cos^2\theta\frac{m^2_{\rho_1}}{m^2_{\rho_2}}\right) - C_{sdH_1}\frac{\sin 2\theta}{2}\left(-1 + \frac{m^2_{\rho_1}}{m^2_{\rho_2}}\right)\right] \times$$
$$\times \left(2\lambda_2 + 2\lambda_3 + \frac{1}{2}\bar{\lambda} + 2\lambda_9 - 5\frac{M}{v_1}\right) . \tag{3.17}$$

The $M$-term in the scalar potential (3.2) does not break any symmetry. One might think then that the ratio $M/v_1$ that appears in eqs. (3.16) can be arbitrarily large and enhance the decay rate. This is however not the case. For very large $M$, the scalar potential would spontaneously break $\mathrm{U}(1)_{EM}$. An even stronger upper bound on $M$ comes from two loop contributions to the neutrino masses. We assumed, for simplicity, that the neutrino masses are dominated by the one loop contributions. This assumption requires that $M/v_1$ is not much larger than the various $\lambda$ couplings. Adding (3.14) to (3.15), and applying the approximation described above eq. (3.15) we find:

$$Y^{(a)}_{\ell m} + Y^{(b)}_{\ell m} \lesssim \frac{m_{\nu_3} M_W}{m^2_{\rho_{\text{light}}}} \frac{m_t^2 m_s}{m^3_{\chi_{\text{light}}}}\left[C_{sdH_1}\left(4 + 3\frac{M_W^2}{m^2_{\chi_{\text{light}}}}\right) + C_{sdH_2}\left(1 + 3\frac{M_W^2}{m^2_{\chi_{\text{light}}}}\right)\right]$$
$$\lesssim 10^2 \frac{m_{\nu_3} M_W}{m^2_{\rho_{\text{light}}}} \frac{m_t^2 m_s}{m^3_{\chi_{\text{light}}}} , \tag{3.18}$$

where $m_{\nu_3}$ is the largest eigenvalue of $M^\nu_{\ell m}$.

We can now compare the CPC rate of the Zee model with the leading CPV one [11]:

$$R^{\text{Zee}}_{\text{CPV}} = \frac{\Gamma_{\text{Zee}}(K_L \to \pi^0 \nu\bar{\nu})}{\Gamma_{\text{CPV}}(K_L \to \pi^0 \nu\bar{\nu})} \lesssim \left(10^2 \frac{m_\nu M_W}{m^2_{\rho_{\text{light}}}} \frac{m_t^2 m_K}{m^3_{\chi_{\text{light}}}}\right)^2$$
$$\approx 10^{-5}\left[\frac{m_\nu}{10\,\text{MeV}}\left(\frac{45\,\text{GeV}}{m_{\rho_{\text{light}}}}\right)^2\left(\frac{45\,\text{GeV}}{m_{\chi_{\text{light}}}}\right)^3\right]^2 . \tag{3.19}$$

Since, very likely, $m_{\nu_\tau} \leq 10\,\text{eV}$ [20] , we expect $R^{\Delta}_{\text{CPV}} \lesssim 10^{-17}$. Recall that this upper bound was obtained using some crude approximations and is expected to be even smaller for an exact calculation.

## 4. Final conclusions

In this work we examined the question of whether the SM CP violating contribution to the $K_L \to \pi^0 \nu\bar{\nu}$ decay is still dominant in the presence of new scalars that induce Majorana masses for neutrinos. We found the following unambiguous answers:

(i) For CPC contributions induced by an SU(2)-triplet scalar, we get:

$$\frac{\Gamma_\Delta(K_L \to \pi^0 \nu\bar{\nu})}{\Gamma_{SM}(K_L \to \pi^0 \nu\bar{\nu})} \lesssim \left(\frac{M_K m_\nu}{m^2_{\xi_{\rm light}}}\right)^2 \lesssim 10^{-11} \, . \tag{4.1}$$

(ii) For CPC contributions that are generated in the Zee model, we get:

$$\frac{\Gamma_{\rm Zee}(K_L \to \pi^0 \nu\bar{\nu})}{\Gamma_{SM}(K_L \to \pi^0 \nu\bar{\nu})} \lesssim \left(10^2 \frac{m_\nu M_W}{m^2_{\rho_{\rm light}}} \frac{m_t^2 m_K}{m^3_{\chi_{\rm light}}}\right)^2 \lesssim 10^{-5} \, . \tag{4.2}$$

In obtaining the final bounds in eqs. (4.1) and (4.2) we used the direct upper bound on $m_{\nu_\tau}$ of order 10 MeV. If we use the cosmological bound, $m_\nu \lesssim 10\,\text{eV}$, then the bounds become stronger by twelve orders of magnitude. It is clear then that the $K_L \to \pi^0 \nu\bar{\nu}$ decay provides a very clean measurement of fundamental, CP violating properties and that it does not probe neutrino masses.


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